BALL MOUNTAIN DAM

JAMAICA, VERMONT

FINAL

REPORT ON THE FIRST THREE CAMPAIGNS

Prepared for
Geotechnical Engineers, Inc.
by
Boston Survey Consultants
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#### EXECUTIVE SUMMARY

This report describes the design, execution and findings of the first three campaigns of Boston Survey Consultants' deformation monitoring program.

The design of the monitoring scheme was completed in early 1986. The installation of the monitoring stations was undertaken during the period May 7 through June 7, 1986. The first series of measurements was completed in July 1986 while the remaining two campaigns took place in September and November of the same year.

A detailed analysis of the observation data has revealed small horizontal movements of a number of the monitored points. Several stations situated on or near the crest edge have undergone horizontal downstream displacements of a few millimetres (lmm - 4mm) while movements of similar magnitude have been detected at two other locations. No significant vertical movements have been identified.

Three further campaigns have been scheduled to take place during 1987. It is anticipated that the additional information gained from these measurements will give a coherent picture of the deformation trends.

## 1. INTRODUCTION

This report describes the design, measurement, adjustment and deformation analysis phases of the first three epochs of the geodetic monitoring scheme. It has been arranged in three parts. This volume includes the written report, Volume II contains the single epoch adjustment results and Volume III includes the deformation analyses. Seven plans accompany the report. The first shows the topography of the dam site as well as the location of the reference and object points. The remaining plans relate to the deformation analysis.

were performed using the Geodetic Network adjustments Network Adjustment (GNA) program while the Localization and Analysis of Deformations (LAD) software was employed for the Both are products of Intergraph deformation analysis. All the statistical tests in the deformation Corporation. analysis have been performed at the 95% level of confidence. Thus a horizontal displacement is "statistically significant at the 95% level of confidence" if the displacement vector extends beyond the perimeter of the 95% confidence region displacement a vertical Similarly, (ellipse). significant at the 95% level of confidence if the vector extends beyond the vertical 95% confidence interval.

# 2. NETWORK DESIGN AND PRE-ANALYSIS

The network design and pre-analysis are interdependent undertakings. The pre-analysis is concerned with the network configuration, the type, number and quality of the observables, the computational requirements and the specification of equipment and observing procedures.

In the case of Ball Mountain Dam, the network configuration is severely constrained by the nature of the site which is characterized by extremely rugged terrain and extensive forest. The final reference network consists of five pillars, a Corps of Engineers disk set in the abutment of the spillway and an additional reference station situated on

top of the intake tower. (see Plan #1). The disk (P6) and tower (1218) stations are treated as object points during the deformation analyses.

In the pre-analysis, consideration was given to the detection of single point displacements both on the dam structure (object points) and in the reference network (reference points). Note that the reference points are presumed to be unstable from one campaign to the next (In the first step of each deformation analysis they are tested for stability at the 95% level of confidence).

A tolerance limit of 3mm at the 95% level of confidence was employed in the pre-analysis. Thus, any single point displacement exceeding 3mm in either the horizontal (x,y) or vertical (z) should be detected as significant at the 95% level of confidence. The pre-analysis was undertaken by a team from the University of New Brunswick led by Dr. Adam Chrzanowski. (Chrzanowski et al., 1985). The following two sections summarize the salient features of the pre-analysis.

# 2.1 THE HORIZONTAL MONITORING SCHEME

In the final design, all possible directions were to be measured from stations Pl, P2, P3, P4 and P5. At the time of the first campaign, it was decided to take additional observations from P6. Four distances were to be measured from P4 to P2, P3, P5 and P6.

The accuracy requirements for the observables are:

- o directions: std. dev. =  $\pm$  +/- 0.5"
- o distances: std. dev. =  $\pm$  5mm  $\pm$  5ppm

Figure 2.1 shows the results of the horizontal preanalysis. The directions are to be measured in 4 sets using an electronic theodolite such as the Wild T2000 or Kern E2. As Chrzanowski et al., (1985) point out this accuracy can be attained only if certain observing precautions are adhered

to. These include shading the theodolite from direct sunlight, using mechanical forced centering for the theodolite and targets, using specially designed targets, and measuring the tilt of the vertical axis. The distances are required to be measured using a suitable electro-optical distance measuring instrument (EODMI). This should be calibrated for zero error and scale and, if necessary, for cyclic error. Appropriate equipment must be employed for measuring the dry bulb temperature and atmospheric pressure.

Ordinary Wild traversing targets are used on the 27 object points and at P6. Special conical, omni-directional targets (Figure 2.2) are employed on the tower and at points P1 through P5. These were designed and produced under contract by the Dept. of Surveying Engineering at the University of New Brunswick.

## 2.2 THE VERTICAL MONITORING SCHEME

The vertical monitoring program requires that all zenith angles from stations Pl through P5 be measured to a standard deviation of 0.7". This can be achieved if 4 sets are observed and if the same precautions are adhered to as those listed in Section 2.1 above. Theodolite and target heights must be measured to an accuracy of at least lmm.

#### 2.3 REFRACTION

Chrzanowski et al., (1985) have emphasized that the ability of the monitoring scheme to detect vertical displacements may be severely degraded by changes in the coefficient of refraction from one campaign to the next. At their suggestion, temperature profile measurements were taken during each of the three campaigns observed to date. A preliminary analysis of these data attests to the severity of this problem (see Appendix IC).

# 2.4 MONUMENTATION

Three kinds of monument were planned (Figures 2.3, 2.4 and 2.5). In the case of the reference pillars, the design reflects the need for an observing platform which it is hoped will remain stable from one epoch to the next. Note the forced-centering socket which ensures precise horizontal relocation of the theodolite. The benchmark provides a reference point for the vertical network. During each campaign, the pillars are wrapped in 5cm thick foam rubber in order to minimize the distortions which may be induced by temperature imbalances.

The slope and crest monuments are designed in such a way that they will adequately represent local movements in their vicinity. To ensure that they are visible from the reference points, the slope monuments protrude approximately lm above the rockfill slope. For the same reason, removable 0.5m extension rods are inserted in the crest monuments during the observing process. In order to ensure precise forced-centering, Wild GRT10 stems were grouted into the tops of these monuments. These match the removable Wild traversing targets, and extension rods.

The substratum associated with each pillar and monument is listed in Table 2.1.

TABLE 2.1
Summary of Pillar and monument Installations

POINT	SUBSTRATUM
P1 P2 P3 P4 P5 P6	bedrock bedrock glacial till bedrock bedrock concrete retaining wall
11,12,13 21,22,23 31,32,33	gravel fill on dam crest
14-19,24-29,34-39	rock fill on downstream slope
1218	on top of concrete intake tower

.P4

directions +/- 0.5"

distances +/- 0.005 +/- 5.0ppm

.P5

.P5

.P6

Figure 2.1 Horizontal pre-analysis after Chrzanowski et al. (1985) Note that the final position of P3 differs slightly from that shown here (see Plan #1)

Single Point Rigid Body Movement Error Ellipses

. P2

100.

Ellipses at  $(1-\alpha)=0.95$ 

0 10.0mm

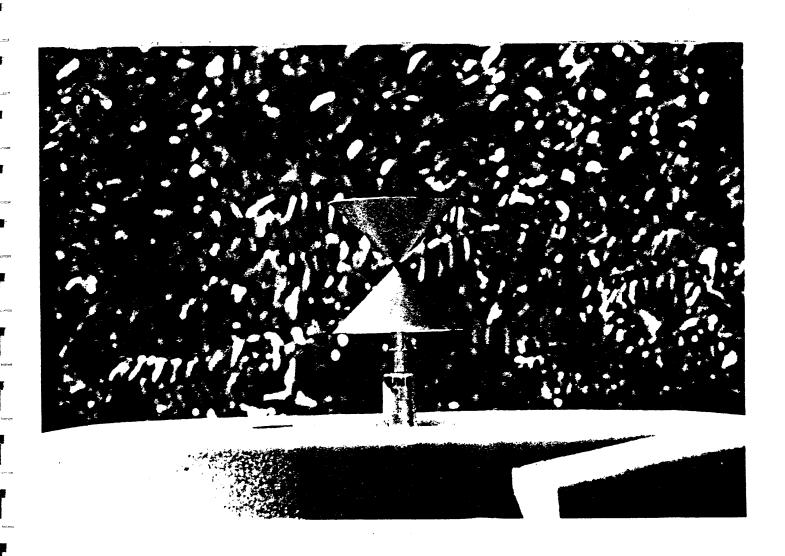


Figure 2.2 The UNB target design.

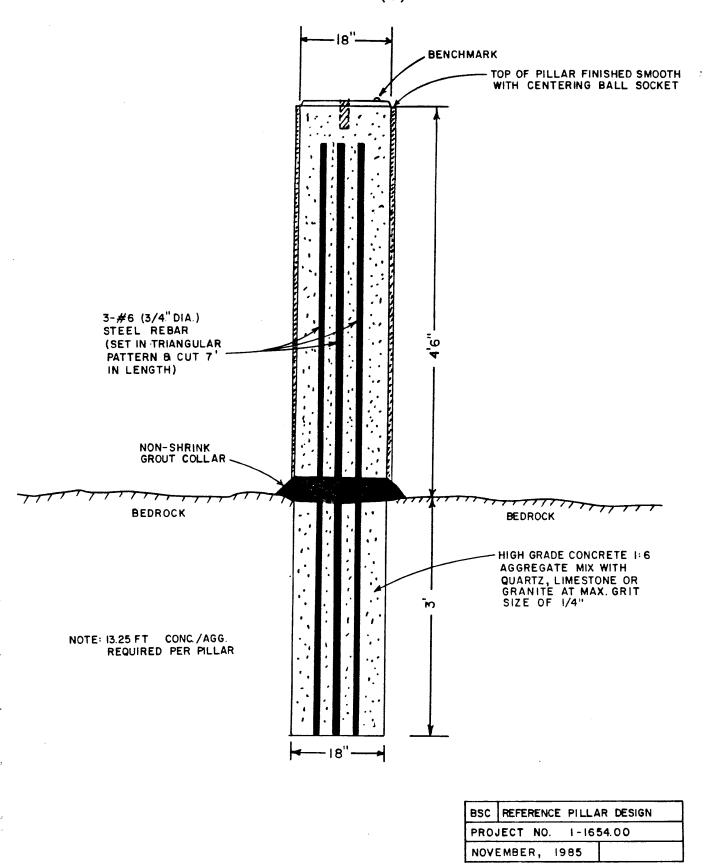


Figure 2.3

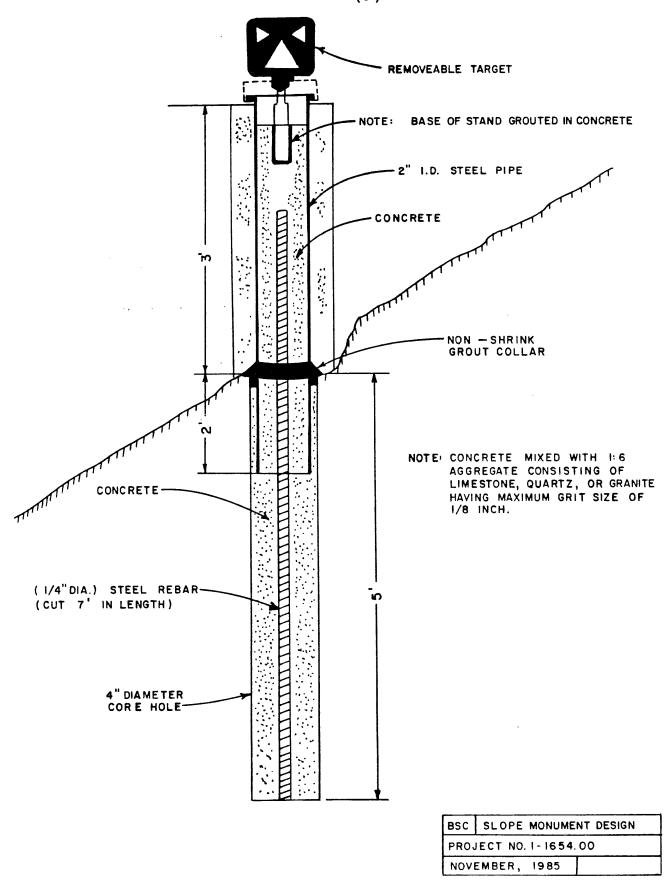
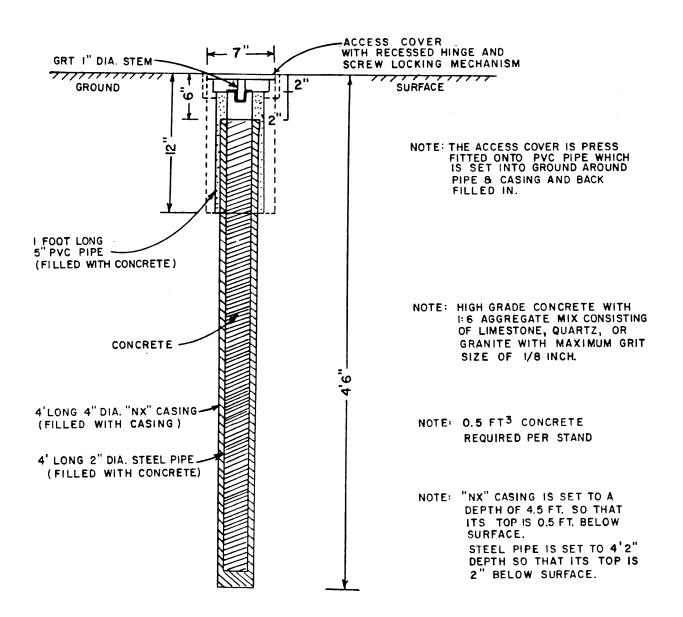


Figure 2.4

# A REMOVEABLE TARGET WILL BE USED DURING THE SURVEY



PROJECT NO. 1-1654.00

NOVEMBER, 1985

Figure 2.5

# 3. OBSERVATION, ADJUSTMENT AND ANALYSIS

The dates of the three epochs are listed in Table 3.1 which also summarizes the prevailing weather conditions.

In the first two campaigns, Wild T2000 and DI4L instruments were used for the angle and distance measurements respectively. A Kern E2/DM502 combination was employed in the third campaign. The two electronic theodolites yielded comparable results and satisfied the specifications. However, the Kern E2 is more appropriate for this kind of work since it can provide precise vertical axis tilt measurements. The Kern DM502 proved to be slightly more precise than the Wild DI4L. This can be ascribed to the availability of high quality calibration data for the former instrument.

Observation data were recorded manually and reduced and checked in the evenings. The inability to perform real-time data validation and on-site station adjustments was a definite handicap. This problem is currently being addressed.

Temperature profile measurements were taken near the theodolite while the angular observations were being made at P3, P4 and P6. Several profiles from the first and second epochs have been analyzed (see Appendix IC). The results confirm the concern expressed in the pre-analysis regarding the severity of the refraction problem.

The horizontal and vertical network adjustments were performed in the office using the GNA software. The salient features of these computations are abstracted in the following sections.

## 3.1 THE HORIZONTAL NETWORK

The GNA results for the three campaigns may be found in Volume II of this report. The "Summary Reports" from each campaign are reproduced in Tables 3.2, 3.3 and 3.4.

In each case the standard deviation of unit weight corroborates the weighting scheme employed in the adjustment. The a priori standard deviation (0.6") used for the directions differs only slightly from the value (0.5") called for in the pre-analysis. In addition, the a priori distance standard deviations (+/- 3mm +/- 4.6 ppm for Epochs 1 and 2 and +/- 2mm +/- 3ppm for Epoch 3) are slightly better than the +/- 5mm +/- 5ppm specified in the pre-analysis.

The relative error ellipses (95% confidence level) indicate that all three epochs have satisfied the specification that the monitoring scheme be capable of detecting a 3mm horizontal movement at the 95% level of confidence.

#### 3.2 THE VERTICAL NETWORK

The GNA results for the three vertical adjustments are contained in Volume II of this report. The "Summary Reports" for each campaign are reproduced in Tables 3.5, 3.6 and 3.7.

Once again the values of the standard deviation of unit weight confirm the a priori weighting scheme. In the first two epochs a standard deviation of 0.5" was used for weighting the zenith angles. A value of 0.6" was employed for the third adjustment.

The absolute 95% confidence intervals vary from 1.4 mm to 3.2 mm. Unfortunately, GNA did not provide the required relative confidence regions. However, reference to the LAD results (Volume III) reveals that the relative 95% confidence intervals for the inter-epoch comparisons varied from 1.7mm to 2.8mm. These results satisfy the specifications.

TABLE 3.1
Synopsis of the three campaigns

Epoch	Dates	Dates Temperature Weathe OF	
VÍ	July 14 15 16 17 18 19 20	75 - 80 75 - 80 75 - 80 - 75 - 85 65 - 75 65 - 75	
II	Sept 23 24 25 26 27 28	65 70 70 70 70 60	overcast, drizzle clear clear, windy in p.m. clear, calm clear overcast
ŢII.	Nov 18 19 20 21 22 23 24	40 20 28 35 28 35 38	overcast, calm clear, windy overcast, windy rain, windy clear, windy clear, light wind partly cloudy, windy

TABLE 3.2

Summary Report: Epoch 1 - horizontal

OBSERVATION TYPE		NUMBER	WEIGHTED SUM OF RESIDUALS SQUARED	REDUNDANCY	STD. DEV. OF UNIT WEIGHT
DIRECTIONS		150	0.659136D+02	0.741045D+02	0.94312
AZIMUTHS		1	0.00000D+00	0.00000D+00	1.00000
HORIZONTAL D	DISTANCE	5	0.123737D+01	0.389550D+01	0.56360
SLOPE DISTAN	ICE .	0	0.00000D+00	0.000000D+00	1.00000
SCALED DISTA	NCE	0	0.00000D+00	0.00000D+0D	1.00000
ZENITH DISTA	NCE	0	0.00000D+00	0.000000D+00	1.00000
HEIGHT DIFFE	RENCE	0	0.000000D+00	0.00000D+00	1.00000
OBSERVED COO	RDINATES	0	0.00000D+00	0.00000D+00	1.00000
COORDINATE D	OIFF.	0	0.00000D+00	0.000000D+00	1.00000
TOTAL		156	0.671510D+02	78	0.92785
STATIONS:	FIXED FREE WEIGHTED	1 32 0			
	TOTAL	33			

CONVERGENCE: ITERATIONS USED 3
TOLERANCE 0.000100
COMPUTATION TIME 38.830 SEC

TABLE 3.3

Summary Report: Epoch 2 - horizontal

OBSERVATION TYPE		NUMBER	WEIGHTED SUM OF RESIDUALS SQUARED	REDUNDANCY	STD. DEV. OF UNIT WEIGHT
DIRECTIONS		139	0.836418D+02	0.690727D+02	1.10042
AZIMUTHS		1	0.00000D+00	0.0000000+00	1.00000
HORIZONTAL D	ISTANCE	4	0.210583D+01	0.292734D+01	0.84816
SLOPE DISTAN	CE	0	0.0000000+00	0.000000D+00	1.00000
SCALED DISTA	NCE	0	0.0000000+00	0.00000D+00	1.00000
ZENITH DISTA	NCE .	0	0.000000D+00	0.0000000+00	1.00000
HEIGHT DIFFE	RENCE	0	0.000000D+0D	0.00000D+00	1.00000
OBSERVED COO	RDINATES	0	0.0000000+00	0.00000D+00	1.00000
COORDINATE D	IFF.	0	0.00000D+00	0.000000D+00	1.00000
TOTAL		144	0.857477D+02	72	1.09130
STATIONS:	FIXED FREE WEIGHTED	1 33 0			
	TOTAL	34			
CONVERGENCE:	ITERATION TOLERANCI COMPUTAT	E	0.000100 99.559 SEC		

TABLE 3.4

Summary Report: Epoch 3 - horizontal

			WEIGHTED SUM OF RESIDUALS		STD. DEV. OF	
OBSERVATION TYPE		NUMBER	SQUARED	REDUNDANCY	UNIT WEIGHT	
DIRECTIONS		135	0.885519D+02	0.651182D+02	1.16613	
AZIMUTHS		1	0.000000D+00	0.00000D+00	1.00000	
HORIZONTAL DI	STANCE	3	0.297731D+01	0.188182D+01	1.25783	
SLOPE DISTANG	CE	0	0.000000D+00	0.00000D+00	1.00000	
SCALED DISTAN	NCE	0	0.000000D+00	0.000000D+00	1.00000	
ZENITH DISTAN	NCE	0	0.00000D+00	0.000000D+00	1.00000	
HEIGHT DIFFER	RENCE	0	0.0000000+00	0.00000D+00	1.00000	
OBSERVED COOF	RDINATES	0	0.00000D+00	0.0000000+00	1.00000	
COORDINATE D	IFF.	0	0.0000000+00	0.00000D+00	1.00000	
TOTAL		139	0.915292D+02	67	1.16881	
STATIONS:	FIXED FREE WEIGHTED	1 33 0				
	TOTAL	34				
CONVERGENCE:	ITERATION TOLERANCE COMPUTATI		2 0.000100 100.453 SEC			

TABLE 3.5

Summary Report: Epoch 1 - vertical

OBSERVATION TYPE		NUMBER	WEIGHTED SUM OF RESIDUALS SQUARED	REDUNDANCY	STD. DEV. OF UNIT WEIGHT
DIRECTIONS		.0	0.000000D+00	0.000000D+00	1.00000
AZIMUTHS		0	0.00000D+00	0.0000000+00	1.00000
HORIZONTAL DI	STANCE	0	0.000000D+00	0.000000D+00	1.00000
SLOPE DISTANCE	E	0	0.0000000+00	0.000000D+00	1.00000
SCALED DISTAN	CE	0	0.00000D+00	0.0000000+00	1.00000
ZENITH DISTAN		132	0.109791D+03	0.919897D+02	1.09248
HEIGHT DIFFER		13	D.337395D+01	0.401033D+01	0.91723
OBSERVED COOR		D	0.0000000+00	0.0000000+00	1.00000
COORDINATE DI		0	0.000000D+00	0.00000D+00	1.00000
TOTAL		145	0.113165D+03	96	1.08572
STATIONS:	FIXED FREE WEIGHTED	1 48 0			
	TOTAL	49			
CONVERGENCE:	ITERATION TOLERANCE COMPUTATI		2 0.000100 14.328 SEC		

TABLE 3.6

Summary Report: Epoch 2 - vertical

			WEIGHTED SUM		STD. DEV. OF
OBSERVATION TYPE		NUMBER	OF RESIDUALS SQUARED	REDUNDANCY	
			0.0000000+00	0.00000D+00	1.00000
DIRECTIONS		_	0.000000D+00	0.000000D+00	1.00000
AZIMUTHS		0	-		1.00000
HORIZONTAL D	ISTANCE	0	0.000000D+00	0.000000D+00	
SLOPE DISTAN	CE	0	0.000000D+00	0.00000D+00	1.00000
SCALED DISTA	NCE	0	0.000000D+00	0.0000000+00	1.00000
ZENITH DISTA	NCE	134	0.885879D+02	0.951584D+02	0.96486
HEIGHT DIFFE	RENCE	12	0.235715D-01	0.841642D+00	0.16735
OBSERVED COO	RDINATES	D	0.0000000+00	0.000000D+00	1.00000
COORDINATE D	DIFF.	0	0.0000000+00	0.00000D+00	1.00000
TOTAL		146	0.886115D+02	96	0.96075
STATIONS:	FIXED FREE WEIGHTED	1 50 0			
	TOTAL	51			

2 0.000100 151.313 SEC

CONVERGENCE: ITERATIONS USED TOLERANCE COMPUTATION TIME

TABLE 3.7

Summary Report: Epoch 3 - vertical

			WEIGHTED SUM OF RESIDUALS		STD. DEV. OF
OBSERVATION TYPE	·	NUMBER	SQUARED	REDUNDANCY	
DIRECTIONS		·O	0.00000D+00	D.000000D+00	1.00000
AZIMUTHS		0	0.00000D+00	0.000000D+00	1.00000
HORIZONTAL DIS	STANCE	0	0.00000D+00	0.000000D+00	1.00000
SLOPE DISTANCE		0	0.000000D+00	0.000000D+00	1.00000
SCALED DISTANG	CE	0	0.000000D+00	0.00000D+00	1.00000
ZENITH DISTAN	CE	120	0.945190D+02	0.800372D+02	1.08671
HEIGHT DIFFER	ENCE	10	0.834129D+00	0.962835D+00	0.93077
OBSERVED COOR	DINATES	0	0.00000D+00	0.0000000+00	1.00000
COORDINATE DI	FF.	0	0.00000D+00	0.0000000+00	1.00000
TOTAL		130	0.953531D+02	81	1.08499
STATIONS:	FIXED FREE	1 50			
	WEIGHTED	0		-	
	TOTAL	51			
CONVERGENCE:	ITERATION TOLERANCE COMPUTATI		0.000100 125.109 SEC		

## 4. DEFORMATION ANALYSIS

## 4.1 HORIZONTAL ANALYSIS

## 4.1.2 EPOCH 1 vs. EPOCH 2

The results for this inter-epoch comparison may be found in Appendix III A. Plan #2 depicts the displacement vectors and their attendant 95% confidence ellipses.

The analysis of the reference stations reveals that point P2 is unstable. The displacement of 2.2mm is in a north-easterly direction. The remaining reference points - P1, P3, P4 and P5 - appear to be stable thereby providing a suitable base for the subsequent deformation analysis.

Five of the <u>27 object</u> points have significant displacements. Two blocks of points were tested for group movement (Table 4.1). The first group containing 17 and 18, has significant X and Y displacements as well as a small significant rotation. The second group which includes all the object points has not moved significantly.

# 4.1.2 EPOCH 2 vs. EPOCH 3

The computations and analysis for this epoch comparison are contained in Appendix III B. The corresponding displacement vectors and their associated 95% confidence regions are presented on plan #3.

The stable base points are Pl, P3, P4 and P5. Once again, the remaining reference point, P2, has been displaced in a north-easterly direction. In this instance, the movement of 3.5mm is somewhat more than the 2.2 mm incurred in the Epoch 1/Epoch 2 interval. The tower (1218) has undergone a significant displacement of 7.1mm in a westerly direction.

Nine (one third) of the object points have been displaced by significant amounts. All of these have a downstream (easterly) component. Inspection of Plan#3 reveals that all but two of the 27 displacement vectors have a downstream component.

There are three fairly clear groupings of points: (1) 32, 33, 34, 35; (2) 17, 18 and (3) 38, 39. Each of these was tested for significant group movement. The results are summarized in Table 4.2. All three have significant X-coordinate (downstream) components. In addition, Group #1 has a significant Y-coordinate (northerly) component. None of the blocks manifests a significant rotation. A fourth group containing all 27 object points was examined. In this case, no translation seems to have occurred although the group does exhibit a very small rotation.

## 4.1.3 EPOCH 1 vs. EPOCH 3

The results for this comparison may be found in Appendix IIIC. The displacement vectors and their associated 95% confidence ellipses are shown on Plan #4. Spanning, as it does, the full July 1986 - November 1986 interval, this inter-epoch analysis yields results which may be viewed as a composite of the previous two analyses (see Sections 4.1.1 and 4.1.2).

Not unexpectedly, P2 manifests a large north-easterly displacement of 5.6mm. The remaining reference points - P1, P3, P4 and P5 - do not have significant displacements. Accordingly, they are used as the base for the ensuing deformation analysis.

Ten of the 27 object points appear to have incurred significant displacements. All of these have a downstream component. Indeed, only one (11) of the object points does not exhibit a downstream trend.

Two blocks of points have been tested for significant group movement. (refer to Table 4.3). The first group, containing points 32, 33, 34, 35 and 36, has a relatively large downstream (X displacement) component and a somewhat smaller northerly (Y displacement) component. For a block of such small size, the rotation is negligible. The second group (points 22, 23 and 24) also has significant X (downstream) and Y (northerly) translations. The attempt to model the combined movement of all object points does not

reveal significant coordinate translations. Although it is statistically significant, the rotation is very small for a block of this extent. Moreover, it is doubtful whether a rotation has any real physical meaning for a non-rigid body such as this.

#### 4.1.4 SUMMARY

It is apparent from the foregoing analysis that most of the deformation activity seems to have occurred in the second inter-epoch interval., i.e., between the September 1986 and November 1986 campaigns. In summary, the following comments may be made with regard to the horizontal deformation analysis:

- 1. Downstream movement of "several millimetres" (1mm 4mm) is manifest in the upper part of line A. This extends from the crest center-line (point 32) down to points 35 and 36 which lie on the bulge.
- 2. Downstream movement of "several millimetres" (1mm 4mm) seems to have occurred in the upper part of line B. This deformation is limited primarily to the immediate neighborhood of the downstream crest edge, i.e., to the vicinity of points 22,23 and 24.
- 3. Points 17 and 18 have undergone significant movements of approximately 2mm-4mm. However, there is no clear directional trend as is evident when examining the vector orientations shown on Plans #2 and #3.
- 4. Some downstream displacement is evident below the bulge in line A (points 38 and 39).
- 5. There seems to be a general downstream trend throughout the set of object points. At present, this "movement" is less than the sensitivity of the monitoring scheme and is not significant at the 95% level of confidence. There is a possibility that this trend is caused by changes in lateral refraction from one campaign to the next.

6. Except for the large north-easterly displacement of point P2, the reference network is stable. The movement of P2 may be ascribed to geological factors (e.g., deformation of the exposed bedrock) or to distortion of the pillar (e.g., concrete shrinkage). An on-site inspection may provide clarification.

TABLE 4.1

Epoch 1 versus Epoch 2

Summary of the group movement analyses - horizontal (Refer to Appendix III A)

Group #	1	2
Stations	17 (3.0) 18 (3.6)	All Object Points
X displacement <sup>1</sup>	-1.0mm	NO
Y displacement <sup>2</sup>	3.8mm	NO
Rotation <sup>3</sup>	-11 arc seconds	NO

- Positive to the east (downstream)
- 2. Positive to the north
- 3. Positive clockwise from north
- Displacement vectors in mm.

TABLE 4.2

Epoch 2 versus Epoch 3:

Summary of the group movement analyses.
(Refer to Appendix IIIB)

Group #	1	2	3	4
Stations	32(2.9) <sup>4</sup> 33(3.7) 34(2.6) 35(2.1)	17(2.7) 18(3.7)	38(1.7) 39(2.6)	All Object Points
X displacement <sup>1</sup>	2.1mm	1.7mm	1.9mm	NO
Y displacement <sup>2</sup>	1.lmm	NO	NO	NO
Rotation <sup>3</sup>	NO	NO	NO -1	arc second

- Positive to the east (downstream)
- 2. Positive to the north
- 3. Positive clockwise from north
- 4. Displacement vectors in mm

TABLE 4.3

Epoch 1 versus Epoch 3

Summary of the group movement analyses - horizontal (Refer to Appendix III C)

Group #	11	2	3
Stations	32 (3.7) <sup>4</sup> 33 (3.4) 34 (2.1) 35 (2.2) 36 (1.4)	22 (1.7) 23 (3.9) 24 (2.0)	All Object Points
X displacement <sup>1</sup>	3.0mm	1.1mm	NO
Y displacement <sup>2</sup>	2.3mm	1.9mm	NO
Rotation <sup>3</sup> +6	arc seconds	NO -1	arc second

- 1. Positive to the east (downstream)
- 2. Positive to the north
- 3. Positive clockwise from north
- 4. Displacement vectors in mm

## 4.2 VERTICAL ANALYSIS

# 4.2.1 EPOCH 1 vs. EPOCH 2

The results for this comparison may be found in Appendix IIID. Plan #5 shows the displacement vectors.

All five reference points (Pl through P5) pass the stability test at the 95% level of confidence. They form a suitable base for the subsequent deformation analysis.

All but two of the 27 object points have negative displacement vectors (Table 4.4). This indicates an apparent downward movement of the points in the Epoch 1 - Epoch 2 interval. The vectors are small, varying in length from 0.02mm to 2.3mm with a mean of 0.9mm. The analysis reveals that none of these displacements is significant at the 95% level of confidence.

## 4.2.2 EPOCH 2 vs EPOCH 3

The results for this analysis are included in Appendix IIIE. The corresponding displacement vectors are shown on Plan #6.

Examination of the base points (P1 through P5) indicates that there is no significant movement of the reference network.

All the object points appear to have undergone substantial positive (upward) vertical displacements (Table 4.4). The movements vary from 0.2mm to 6.2mm with a mean value of 2.7mm. Twenty one (78%) of the 27 displacements are significant. An attempt was made to model the object point movement. Considered together as a single group, the 27 points have a positive translation of 4.3mm.

## 4.2.3 EPOCH 1 vs. EPOCH 3

The results for this comparison may be found in Appendix IIIF. Plan #7 shows the displacement vectors. Once again, this epoch comparison should be viewed as a composite of the previous two analyses (see Sections 4.2.1 and 4.2.2).

The stable point analysis reveals no significant movement of the reference points. All 27 object points manifest positive (upward) displacements which vary from 0.2mm to 4.3mm with a mean of 1.9mm (Table 4.4). Eleven (41%) displacements are significant. The group analysis, which includes all object points, reveals a positive (upward) translation of 3.4mm.

#### 4.2.4 SUMMARY

The inter-epoch comparisons reveal very obvious trends in the displacement vectors (Table 4.4). These may be explained by the effect of changes in the coefficient of refraction from one campaign to the next. This problem is addressed in some depth in Appendix IC. At present - considering the 4 month interval between Epochs 1 and 3 - the expected size of the point displacements is of the same order of magnitude as the systematic refraction error. Therefore, it is not possible to discriminate between real vertical movements and the apparent displacements caused by refraction.

Owing to the fact that the rays linking the reference stations have ample ground clearances, they are less affected by changes in refraction than are the rays between the reference and object points which generally graze close to the surface. This is borne out by the results of the base point analyses which reveal that all five reference stations have remained stable at the 95% level of confidence.

TABLE 4.4
Single Point Displacements (vertical network analysis)

Point	Epoch 1	Epoch 2	Epoch 1
	to	to	to
	Epoch 2	Epoch 3	Epoch 3
P1	-1.5	0.6	-0.9
P2	0.3	0.06	0.4
P3	1.6	-1.0	0.5
P4	-0.8	-0.9	-1.6
P5	0.4	1.2	1.6
P6	-0.8	0.2	-0.6
31 32 33 34 35 36 37 38 39	0.02 -0.7 -1.0 -1.1 -0.3 -0.7 -0.3 -0.8 -2.3	3.5* 3.4* 2.0* 2.0* 1.4 1.6 3.1* 3.2* 2.7*	3.6* 2.7* 1.0 0.9 1.1 0.8 2.8 2.4 3.4*
21 22 23 24 25 26 27 28 29	-0.4 -0.9 -0.7 -1.5 -0.8 -0.3 -0.9 -1.9	2.9* 3.3* 3.0* 1.7 2.9* 4.7* 2.5 1.8 6.2*	2.5* 2.9* 2.1 1.0 1.4 3.9* 2.1* 0.9 4.3*
11	-1.5	3.4* 3.7* 2.6* 1.6 2.3* 2.0* 2.3* 2.7* 2.9*	1.9
12	-0.4		3.2*
13	-0.7		1.9
14	-1.4		0.2
15	-1.4		0.9
16	-0.6		1.4
17	0.06		2.4*
18	-1.6		1.1

<sup>\* -</sup> Displacement is significant at the 95% level of confidence (significance level = 0.05)

#### 5. CONCLUSIONS

In general, the outcome has confirmed the relevance of the pre-analysis. The horizontal deformation results are pleasing. The vertical analysis has suffered from the adverse influence of atmospheric refraction. Since the refraction errors vary depending on the vertical air temperature gradients, spurious vertical displacements of as much as 5mm may occur when comparing data from winter and summer campaigns. Estimates for the refraction errors may be obtained by measuring vertical temperature gradients and/or by running a few selected lines of precise geometric leveling (See Appendix IC). However, for several reasons, this problem may not be all that intractable:

- o The refraction error may be expected to be similar for campaigns having similar prevailing temperature conditions. For example, the July 1986 vs July 1987 comparison should be largely free from the influence of refraction.
- o Over longer time periods the point displacements may be significantly greater than the average magnitude of the refraction error.

The analysis has shown that some deformation is evident in the structure, particularly along the crest edge near the spillway. This may be confirmed by the analysis of the slope inclinometer data and by the remaining campaigns of the geodetic monitoring program. The results from the first three epochs have shown that geodetic techniques are well suited to monitoring programs of this kind.

# 6. BIBLIOGRAPHY

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Department of Surveying Engineering, University of New Brunswick, Canada.

# APPENDIX IA

# Summary of Daily Activities

The following tabulations have been abstracted from Boston Survey Consultants' "Daily Chief Reports":

# IA.1 Epoch 1

DATE	WEATHER	ACTIVITY
7/14/86	clear 75°F - 80°F	travel to site, cut lines wrapped pillars, set targets
7/15/86	clear, windy, 75°F - 80°F	direction obs. at Pl,P4 and P5
7/16/86	clear, hazy, 75°F - 80°F	direction obs at P4 distance obs P6 to P1,P2,P4 and P5
7/17/86		direction obs at P3
7/18/86	clear, 75°F - 85°F	direction obs at P2
7/19/86	overcast, windy, 65°F - 75°F	direction obs at P6
7/20/86	windy, light rain 65 <sup>0</sup> F - 75 <sup>0</sup> F	repeated some dir obs at P3 repeated some dir obs at P6 measured distance P6-P3 travel to Boston.

IA.2 Epoch 2

DATE	WEATHER	ACTIVITY
9/23/86	overcast, light rain, 65°F	travel to site, site preparation, install PVC sleeves, install bench marks on two pillars
9/24/86	fog in a.m. clear p.m. 70°F	install two pillar bench marks install water target (1218), paint PVC tubes, direction obs at P6
9/25/86	clear, calm a.m. wind in p.m. $70^{\circ}$ F	install one pillar bench mark, direction obs at Pl, direction obs at P2
9/26/86	clear,calm 70°F	direction obs. at P3 and P5, calibrated NA2 level
9/27/86	clear 70°F	direction obs. at P4, repeated dir obs at P6
9/28/86	overcast, 60°F	distance measurements P4 to P1,P2, P3 and P5, some additional dir obs at P6, level from NGVD disc to P1, winterize crest and pillar monu- ments, clean up site, travel to Boston

IA.3 Epoch 3

DATE	WEATHER	ACTIVITY
11/18/86	overcast -5°C	travel to site, direction obs. at Pl and P5, set out crest targets and tripod at P6.
11/19/86	windy -6°C	prepared all pillars, P6 and slope monuments, dir obs from P4
11/20/86	overcast, windy -2°C	direction obs. at P6 and P3
11/21/86	rain, snow, wind 2°C	no field work owing to severe weather conditions, some data reductions done.
11/22/86	clear, windy -2°C	direction obs at P3 and P2
11/23/86	clear, windy 30C	direction obs. at P2 and P5, distance measurements P4, to P1, P3, P5, P6.
11/24/86	fog, windy 30C	direction obs. at P6, distance measurements P6 to P1,P2, travel to Boston

# APPENDIX IB

Report No. 2

## THE BSC GROUP

# SAFETY MEETING AT BALL MOUNTAIN DAM

TO: Safety Office, NED FROM: Division Manager

Date Held: July 14, 1986

Time: 08:00 hours

Safety meeting was held this date for the following BSC Group personnel:

Clark R. Donkin Mark W. Rohde L. Jeff Lowell

Conducted by: Kevin Hanley

Subjects discussed included:

Accident Prevention Individual Protective Equipment Prevention of Falls

Report No. 1 Page 2 of 2

#### THE BSC GROUP

## SAFETY MEETING AT BALL MOUNTAIN DAM

Safety Office, NED TO: FROM: Division Manager

Date Held: July 14, 1986

Time: 08:00 hours

Total on-site exposure hours for BSC Group personnel:

July 14, through July 20, 1986:

81.0 manhours Clark R. Donkin Mark W. Rohde 81.0 manhours L. Jeff Lowell 81.0 manhours

Signature: Mein Aformley
The BSC Group / Division Manager

Report No. 2 Page 1 of 2

#### THE BSC GROUP

# SAFETY MEETING AT BALL MOUNTAIN DAM

TO: Safety Office, NED FROM: Division Manager

Date Held: August 25, 1986

Time: 11:00 hours

Safety meeting was held this date for the following BSC Group personnel:

Clark R. Donkin
Mark W. Rohde
L. Jeff Lowell
W.J. Trevor Greening

Conducted by: Kevin Hanley

Subjects discussed included:

Accident Prevention
Individual Protective Equipment
Prevention of Falls

Report No. 2 Page 2 of 2

#### THE BSC GROUP

## SAFETY MEETING AT BALL MOUNTAIN DAM

Safety Office, NED TO: FROM: Division Manager

Date Held: August 25, 1986

Time: 11:00 hours

Total on-site exposure hours for BSC Group personnel:

# September 23, through September 28, 1986:

Clark R. Donkin	53.0 manhours
Mark W. Rohde	53.0 manhours
L. Jeff Lowell	53.0 manhours
W.J. Trevor Greening	32.5 manhours
Kevin Hanley	5.0 manhours

Signature: Mining Aanley
The BSC Group / Division Manager

Report No. 3
Page 1 of 2

#### THE BSC GROUP

# SAFETY MEETING AT BALL MOUNTAIN DAM

TO: Safety Office, NED FROM: Division Manager

Date Held: November 18, 1986

Time: 08:30 hours

Safety meeting was held this date for the following BSC Group personnel:

Clark R. Donkin
Mark W. Rohde
L. Jeff Lowell
W.J. Trevor Greening

Conducted by: Kevin Hanley

Subjects discussed included:

Accident Prevention
Individual Protective Equipment
Prevention of Falls

Report No. Page 2 of 2

#### THE BSC GROUP

# SAFETY MEETING AT BALL MOUNTAIN DAM

TO: Safety Office, NED FROM: Division Manager

Date Held: November 18, 1986

Time: 08:30 hours

Total on-site exposure hours for BSC Group personnel:

# July 14, through July 20, 1986:

52.0 manhours Clark R. Donkin 52.0 manhours Mark W. Rohde 26.5 manhours L. Jeff Lowell W.J. Trevor Greening 26.5 manhours

Signature: Merin Hanley
The BSC Group / Division Manager

#### APPENDIX IC

Ball Mountain Dam
An Assessment of the Effects
of Atmospheric Refraction

#### INTRODUCTION

In the Preanalysis of the monitoring survey it was pointed out that changes in the coefficient of refraction may adversely affect the results of the trigonometric levelling at Ball Mountain Dam. It was suggested that the problem might be ameliorated by measuring vertical temperature gradients/profiles during each campaign. Accordingly, several trial profile measurements were undertaken during the July 1986 and September 1986 epochs. A preliminary analysis of these data indicates that the influence of refraction requires further and continued attention.

#### THEORETICAL BACKGROUND

Kukkamaki (1938, 1939a, 1939b) proposed that vertical temperature gradients and hence refraction corrections could be estimated from observed vertical temperature profiles by means of a simple mathematical model:

$$T = a + bh^{C}$$
 (1)

where a, b and c are constants for a particular profile. T is the mean temperature at height h above the ground. By measuring several (at least 3) temperatures at different heights it is possible to solve for the values of a, b and c. If redundant measurements are made then estimates can be obtained using a least squares adjustment.

The mean vertical temperature gradient is obtained by differentiation of equation (1):

$$\frac{dT}{dh} = bch^{c-1} \tag{2}$$

In accordance with the free convection theory, many authors (e.g. Fraser, 1977; Holdahl, 1982) set c=-1/3. This assumption is valid during typical unstable daytime conditions.

The coefficient of refraction may be computed using the expression (see e.g., Greening, 1985):

$$k = 78.83 \frac{PR}{T^2} [0.0342 + \frac{dT}{dh}] 10^{-6}$$
 (3)

where P is the atmospheric pressure [mb],

R is the radius of curvature of the earth [m], and

T is the mean atmospheric temperature [K].

In equation (3), the gradient dT/dh is the dominant term. The value of k is rather insensitive to assumptions made with regard to the atmospheric pressure (P) and temperature (T).

Finally, the total refraction error in a particular sighting can be evaluated by numerical integration along the optical path (Angus-Leppan, 1971; 1979):

Ref = 
$$\frac{1}{R} \left\{ \frac{s_1}{2} (k^1 S + k_2 [S-s_1]) + \frac{s_2}{2} (k_2 [S-s_1] + k_3 [S-s_1-s_2]) + \dots + \frac{s_n}{2} (k_n [S-s_1-\dots-s_n-1] + 0) \right\}$$
 (4)

R is the radius of curvature of the earth,  $s_1$ ,  $s_2$ , ...,  $s_n$  are successive subsections of the total distance S and  $k_1$ ,  $k_2$ , ...,  $k_n$  are the corresponding coefficients of refraction.

The use of equation (4) pre-supposes a fairly detailed knowledge of the terrain profile and temperature stratification along the line of sight. In the following section, equations (1) through (4) are employed to evaluate the significance of the refraction error in the heighting of one of the object points on the dam wall. These computations have been made to assess the seriousness of the refraction phenomenon. They are not intended for the application of corrections.

#### RESULTS

Table 1 shows the temperature measurements for four profiles taken from the July and September campaigns. For each data set, a least squares adjustment provided estimates of the coefficients a and b (Table 2). the value of c=-1/3 was held fixed. Graphs of the computed profiles appear in Figure 1 through 4. Equations (2) and (3) were used for computing the vertical temperature gradients and corresponding values of k (Figures 5 through 8).

When  $dT/dh < 0.0342~Cm^{-1}$ , then k < 0 and the curvature of the optical path is convex to the ground. In this case the object points will appear to be lower than their true positions (Figure 11). If  $dT/dh > 0.0342~Cm^{-1}$ , then k > 0 and points will appear higher than their true locations. The former situation can be expected to occur during typical warm summer days when the heat flux is upward out of the ground. In winter months and/or at night the latter may occur.

In order to assess the problem, refraction errors were computed for the height differences P3-28 and P6-28. Figure 9 shows the horizontal positions of these stations. The terrain profiles are presented in Figure 10 and the refraction error computations are summarized in Table 3.

It is somewhat surprising that the computed refraction errors agree so well. In general, variations of "several millimetres" may be expected to occur. However, the outcome does emphasize the systematic nature of the phenomenon. During any particular campaign, a trend may occur throughout the set of object points. Unfortunately this systematic effect may change seasonally. For example, in summertime, when strong negative temperature gradients predominate, the average refraction error may lie in the range -10mm to -3mm. On the other hand, during winter the near surface gradients may be positive in which case the refraction error is positive. This seasonal variation would be manifest as an apparent upward movement of the object points from summer to winter. The opposite would occur in the winter - summer interval.

## CONCLUSIONS AND RECOMMENDATIONS

It is clear that the refraction problem requires further attention. If the effect is ignored, it may be extremely difficult to discriminate between spurious refraction induced displacements and the real vertical motions of the object points.

The following recommendations are made:

- The process of collecting vertical temperature profile information should be fully implemented.
- Resources should be allocated to the analysis of the temperature data.
- 3. Resources should be made available for precise geometric levelling between certain network points as a means of verifying the systematic refraction error component.

With regard to recommendation 3 above, it is suggested that the following loop be measured: P4-31-21-11-P6-15-25-35-P4. This will entail the observation, under very difficult conditions, of approximately 35-40 setups. The levelling would require one extra field day per epoch while the analysis of the temperature profile and levelling data will necessitate a further 4 days' office work.

WTG/gh

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TABLE 1 SUMMARY OF TEMPERATURE PROFILE MEASUREMENTS

	S dgm c			
REMARKS	middle of crest, clear, sunny, 15 mph winds, Pl and P5 occupied	P3 occupied gradients in bush clowdy / calm	P3 occupied, clowdy / calm	P6 occupied, Sunny / calm
u.	26.0	26.2	21.0	25.2
ш	25.1	26.2	20.8	24.4
۵	24.5	25.6 25.4 26.2 26.2	15.2 16.9 16.8 18.7 20.8 21.0	22.9 23.2 24.4 25.2
U	23.5	25.4	16.8	22.9
æ	22.8		16.9	22.5
∢	22.5	24.7	15.2	22.1
TIME	13:00	11:30	11:30	14:00
DATE	1986 - 0715	1986 - 0717	1986 - 0727	1986 - 0924

PROBE HEIGHTS A 4.0 B 3.0 C 2.0 D 1.2 E 0.6 F 0.3

WTGTAB1.20

TABLE 2

SUMMARY

LEAST SQUARES SOLUTION FOR COEFFICIENTS OF THE TEMPERATURE PROFILE

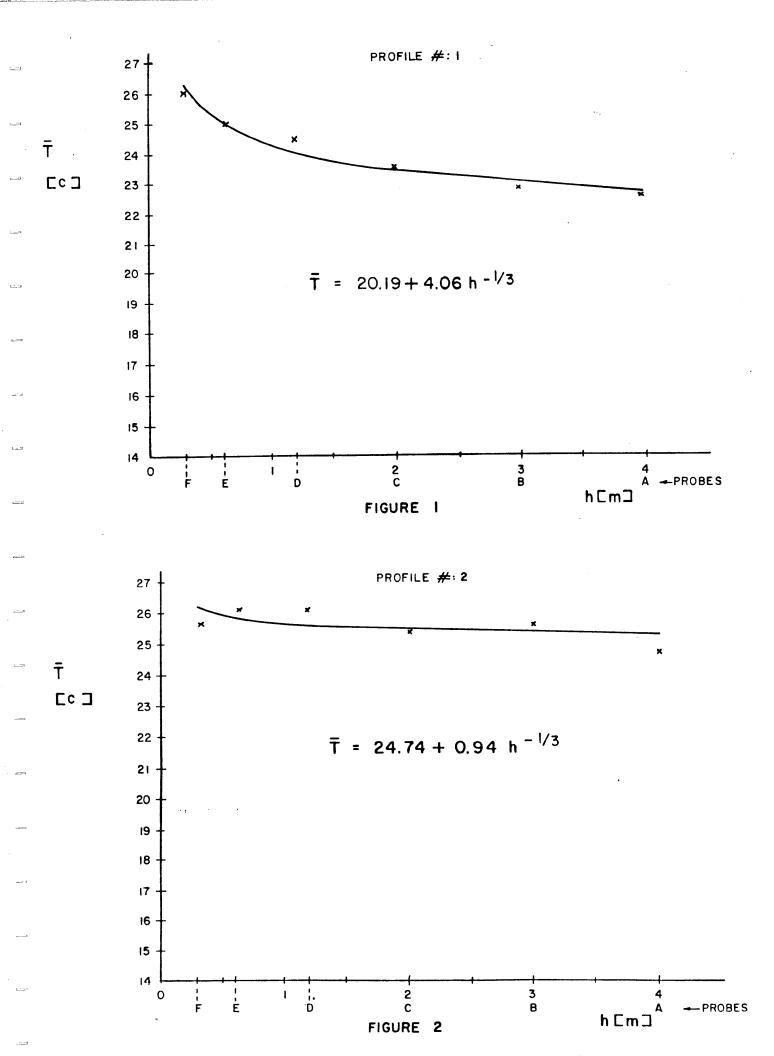
4	19.968	3.571	0.61	0.83	0.83	-0.95	0.12	-0.06	-0.10	0.13	-0.20	01.0
m	11.829	6.697	3.52	4.79	4.78	-0.95	0.85	-0.43	0.34	-0.57	-1.03	0.83
8	24.739	0.935	2.10	2.85	2.84	-0.95	0.63	-0.21	0.08	-0.58	-0.35	0.44
-	20.188	4.057	1.30	1.772	1.767	-0.95	0.24	0.20	-0.09	-0.50	-0.10	0.25
	ĸ	۵					>	>	>	>	>	>

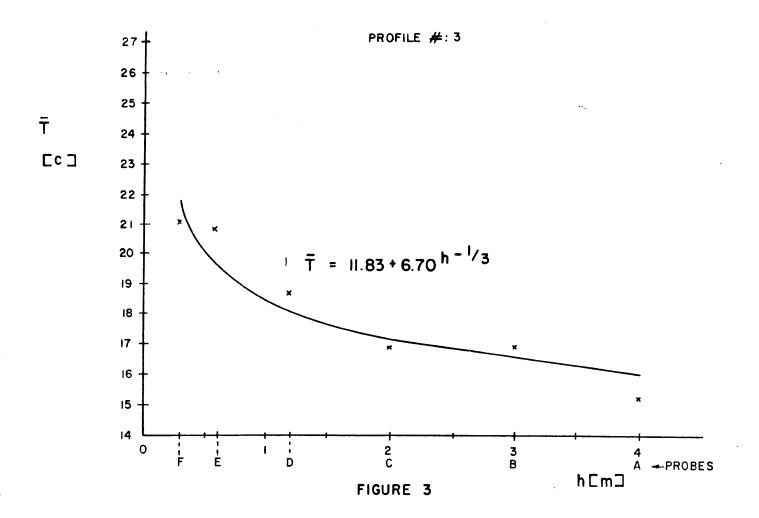
RESIDUALS:

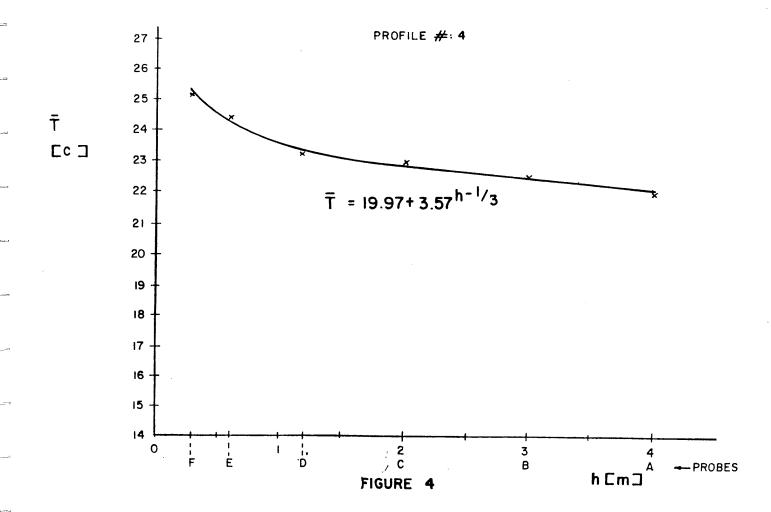
TABLE 3

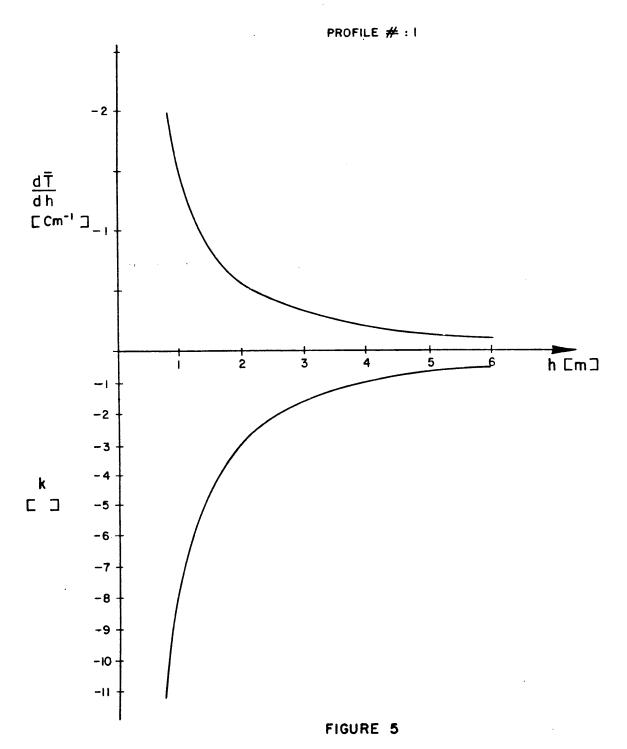
# SUMMARY OF REFRACTION ERROR COMPUTATIONS

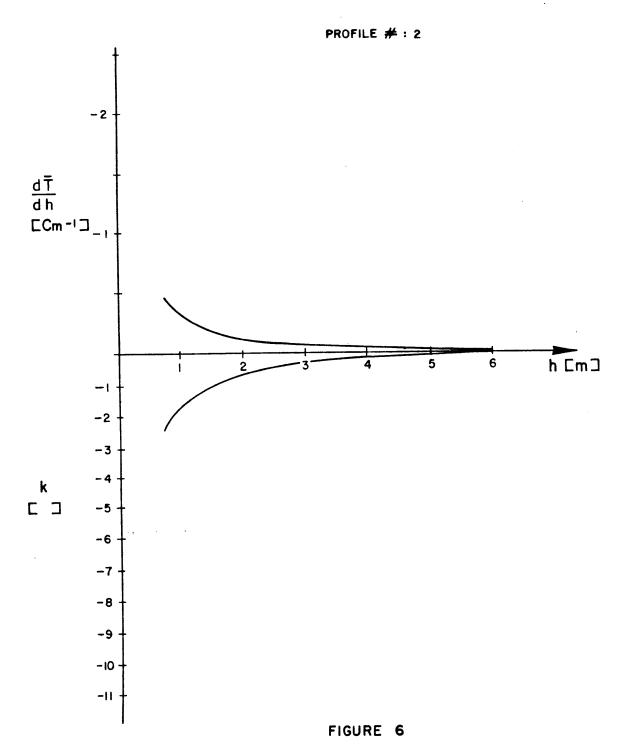
LINE P3 - B8		•			
SEGMENT *	-	2	ო	4	
SEGMENT LENGTH	50m	47	33	32	
ACCUMULATED DISTANCE	50m	97	130	162	
RAY CLEARANCE	1.8m	11.5	22.3	11.0	from figure
POINT VALUE OF K	-5.8	-0.3	-0.01	-0.3	from figure
REFRACTION ERROR	-3.82mm	-0.13	-0.03	-0.02 TC	TOTAL: -4.0mm
LINE P6 - B8					
SEGMENT *	<b>-</b>	8	ო	4	
. SEGMENT LENGTH	35m	35	35	36	
ACCUMULATED DISTANCE	35	70	105	141	
RAY CLEARANCE	. 5a	2.1	2.7	1.8	from figure
POINT OF VALUE OF K	-3.8	2.3	-1.6	-2.9	from figure
REFRACTION ERROR	-2.14mm	86.01	-0.60	-0.29	(

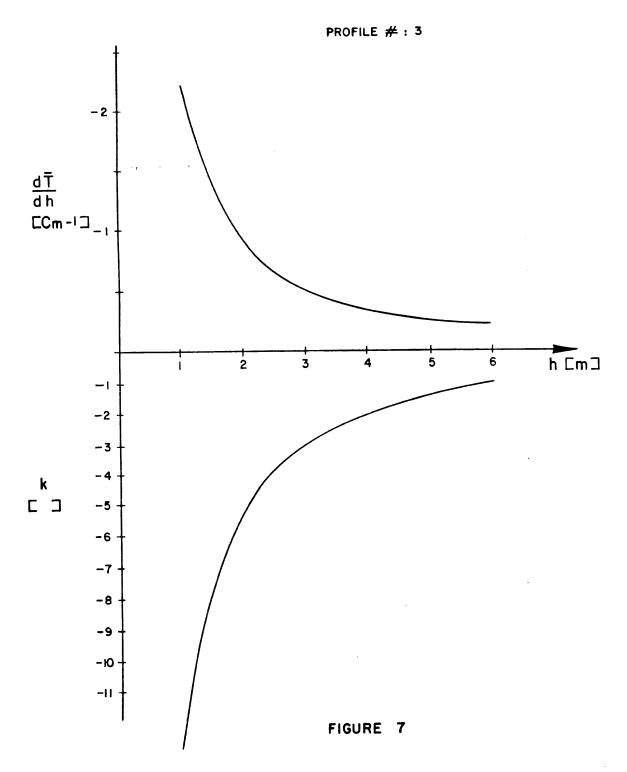


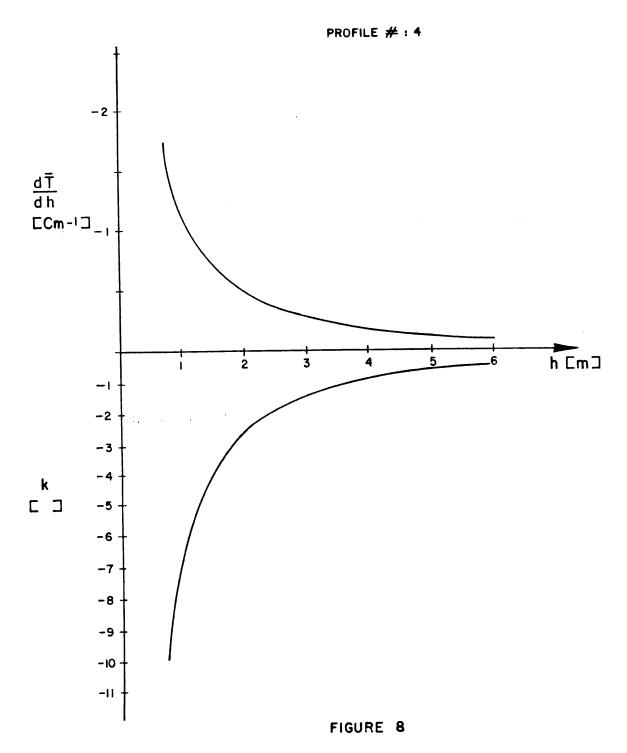




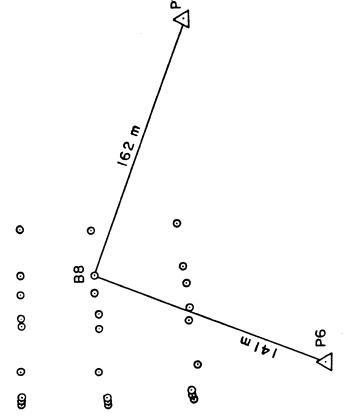






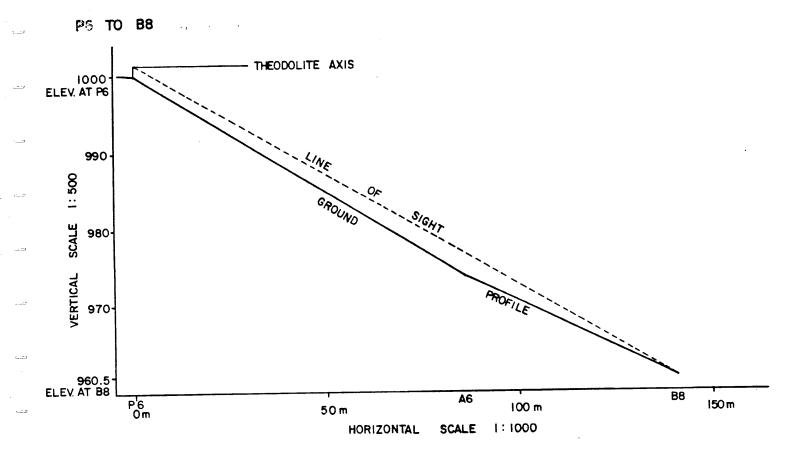


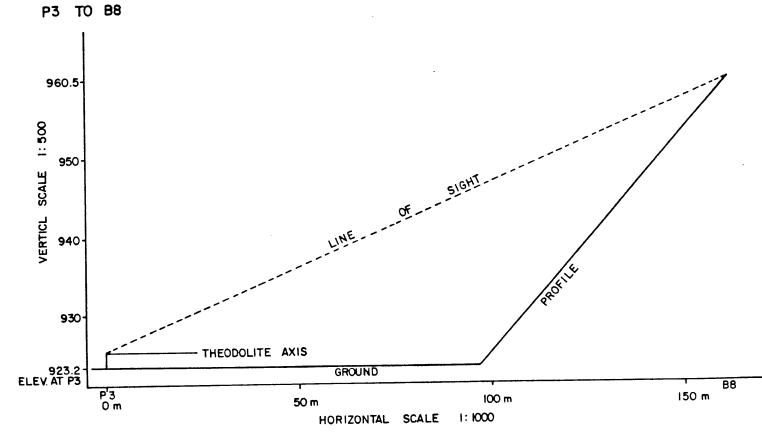
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FIQURE 10

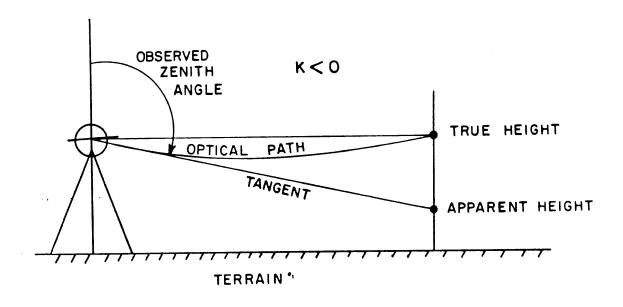


FIGURE II

THE EFFECT OF A NEGATIVE COEFFICIENT OF REFRACTION